

1-5 Solving Inequalities

Properties of Order $a, b, c \in R$

Addition

If $a > b$, then $a + c > b + c$ If $a < b$, then $a + c < b + c$

Subtraction

If $a > b$, then $a - c > b - c$ If $a < b$, then $a - c < b - c$

$$a, b, c \in R$$

Multiplication


If $a > b$, and if c is positive, then $a \cdot c > b \cdot c$ If $a < b$, and if c is positive, then $a \cdot c < b \cdot c$ If $a > b$, and if c is negative, then $a \cdot c < b \cdot c$ If $a < b$, and if c is negative, then $a \cdot c > b \cdot c$

Division

If $a > b$, and if c is positive, then $\frac{a}{c} > \frac{b}{c}$ If $a < b$, and if c is positive, then $\frac{a}{c} < \frac{b}{c}$ If $a > b$, and if c is negative, then $\frac{a}{c} < \frac{b}{c}$ If $a < b$, and if c is negative, then $\frac{a}{c} > \frac{b}{c}$

Examples

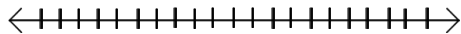
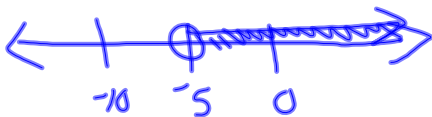
ex 1:

$$\begin{aligned}
 3(x - 5) &< 30 && \rightarrow x - 5 < 10 \\
 &&& x < 15 \\
 3x - 15 &< 30 \\
 3x &< 45 \\
 x &< 15
 \end{aligned}$$


ex 2:

$$5 - 12y < 65$$

$$\begin{aligned} -12y &< 60 & * \div \text{neg.} \\ y &> -5 \end{aligned}$$

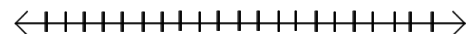


ex 3:

$$4 \left[\frac{3(2-5x)}{4} - 1 \leq 5 - \frac{3(3-2x)}{2} \right]$$

$$\begin{aligned} 3(2-5x) - 4 &\leq 20 - 6(3-2x) \\ 6-15x-4 &\leq 20-18+12x \\ 2-15x &\leq 2+12x \end{aligned}$$

$$\begin{aligned} 0 &\leq 27x \\ 0 &\leq x \\ x &\geq 0 \end{aligned}$$



Notation

set-builder notation

$$\text{ex: } \{x \mid x < 15\}$$

ex 2: $\{y \mid y > -5\}$ "the set of all x such that $x < 15$ "

ex 3: $\{x \mid x \geq 0\}$

interval notation

$$+\infty - \infty \text{ (open) } [\text{closed}] \geq \leq$$

ex 1: $(-\infty, 15)$ ex 3: $[0, +\infty)$

ex 2: $(-5, +\infty)$